AD-A072 936 SOUTHERN METHODIST UNIV DALLAS TEX DEPT OF STATISTICS F/G 12/1

ONE-SIDED TOLERANCE LIMITS FOR A BROAD CLASS OF LIFETIME DISTRI--ETC(U)

JUL 79 W A WOODWARD, W H FRAWLEY N00014-75-C-0439

UNCLASSIFIED TR-135 NL

1 OF 1

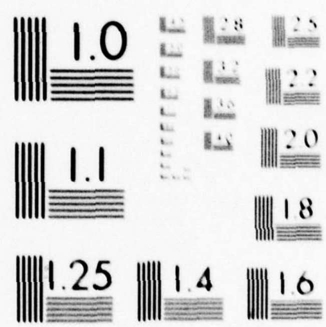
AD A072936

END

DATE FILMED

9-79

DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(12) LEVEL II

AD A072936

SOUTHERN METHODIST UNIVERSITY

DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited

DEPARTMENT OF STATISTICS

DALLAS, TEXAS 75275

DDC
RECEIVED
AUG 21 1979
B

79 08 20 035

(12) LEVEL II

(9) Technical rept.,

(6)

ONE-SIDED TOLERANCE LIMITS FOR A BROAD CLASS OF LIFETIME DISTRIBUTIONS WITH APPLICATIONS TO DATA OF LIMITED ACCURACY.

(10) Wayne A. Woodward
Department of Statistics
Southern Methodist University
Dallas, Texas 75275

William H. Frawley
E-Systems, Incorporated
Greenville, Texas 75401

(14) TR-
Technical Report No. 135
Department of Statistics ONR Contract
(11) 25 July 1979

(12) 27p.

(15)

Research Sponsored by the Office of Naval Research
Contract N00014-75-C-0439
Project NR 042-280

DIVISION OF MATHEMATICAL SCIENCES
Department of Statistics
Southern Methodist University
Dallas, Texas 75275

One-Sided Tolerance Limits for a Broad Class of Lifetime Distributions with Applications to Data of Limited Accuracy

ABSTRACT

Addressed is the problem of determining a one-sided tolerance limit for a population possessing a distribution belonging to a broad class of lifetime distributions. A new implementation of existing general theory is given and contrasted with an earlier utilization of that theory. General guidelines are given for deciding which implementation to use. A method for adjusting for the accuracy of the measuring device is discussed and illustrated with an actual example.

One-Sided Tolerance Limits for a Broad Class of Lifetime Distributions with Applications to Data of Limited Accuracy

Wayne A. Woodward
Department of Statistics
Southern Methodist University
Dallas, Texas 75275

and

William H. Frawley
E-Systems, Incorporated
Greenville, Texas 75401

Introduction

Sometimes a specification of a manufactured product must be stated in terms of an upper or lower tolerance limit for the attribute of the item produced. For example, a manufacturer might state the probability that a certain portion of mechanical components will attain at least a given lifetime. Or a company may claim with a certain confidence that virtually all (stated as a proportion) of its safety devices will trigger before a dangerous condition exists.

The specific theory that is applied to provide this information will depend on what is known about the underlying lifetime density function. Often, sample data is either scanty or else indicates that it would be unlikely that the assumptions necessary to employ parametric procedures would be valid. In either of these situations it is necessary to turn to distribution-free methods in order to determine the desired tolerance limits.

Research for this paper was partially supported by ONR Contract No. N00014-75-C-0439. W. A. Woodward is Assistant Professor of Statistics and Manager of the Statistical Research Laboratory at Southern Methodist University. W. H. Frawley is Supervisor of Analytic Services at E-Systems.

However, for a given confidence level γ and content 1-P the traditional non-parametric tolerance limit (see [5], pp. 491-492) obtained from using one of the sample order statistics requires a minimum sample size; and, for any of a multitude of reasons, it may not be possible to obtain this minimum sample size.

Hanson and Koopmans [2] developed a technique for calculating one-sided tolerance limtis for a broad class of lifetime distributions (see Appendix A). They also implemented their theory by publishing tables of factors which can be used to calculate tolerance limits using two adjacent order statistics. In practice, even though the underlying distribution is continuous, the measuring instruments yield observations to only a certain degree of accuracy. This can prove troublesome when the degree of imprecision is relatively large, especially when tie observations are likely. We investigate the problem of applying the Hanson and Koopmans results in these cases. As a result of this investigation another implementation of the theory in [2] which depends on the smallest and largest order statistics is discussed.

The Hanson and Koopmans Method

The statistic L will be called a lower 1-P content tolerance limit with confidence level γ if the probability is at least γ that at least proportion 1-P of the population falls above L. Upper tolerance limits are defined in the analogous manner. In the discussion to follow it will be assumed that the underlying lifetime distribution is such that either upper or lower tolerance limits may be found using the method of Hanson and Koopmans see (Appendix A). Their lower tolerance limit is

$$L = Y_{k+j+1} - b(Y_{k+j+1} - Y_{k+1}) \tag{1}$$

while their upper tolerance limit is

$$U = Y_{n-k-j} + b(Y_{n-k} - Y_{n-k-j}) \tag{2}$$

where Y_m is the m^{th} order statistic from a sample of size n. For a discussion concerning the evaluation of the constant b see Appendix A and and [2]. The tables presented by these authors were for the case $j = 1$ which corresponds to the use of consecutive order statistics in the tolerance limits, usually the two smallest or two largest for lower and upper limits, respectively. The lower tolerance limit using adjacent order statistics and $k = 0$ is

$$L_H = Y_2 - b_H(Y_2 - Y_1) \tag{3}$$

whereas the corresponding upper tolerance limit is

$$U_H = Y_{n-1} + b_H(Y_n - Y_{n-1}) \tag{4}$$

where b_H is tabulated in [2].

We will investigate the application of these tolerance limits in assessing the strength of steel pipe. The strength of steel pipe is measured by the amount of pressure which must be exerted before the pipe collapses. The pressure is increased by increments of 100 pounds until casing collapse occurs. Government specifications have set a catalog minimum for each grade of pipe. If a company is to advertise a pipe as being of a certain grade, then it must be able to show that the pipe will withstand a pressure at least as great as the catalog minimum. The procedure employed is to take a sample of the pipes, usually of size less than 100 from a particular grade because of the expense of testing, and determine whether or not the catalog minimum is above or below the lower tolerance limit with $1-P = .995$ and $\gamma = .95$. Due to the measuring technique the data is collected only to the nearest 100 pounds and as Table 1 indicates ties occur frequently.

(Table 1 here)

Direct application of (3) yields $L_H = 6000 - b_H(6000-6000) = 6000$. In fact if $Y_1 = Y_2$, then $L_H = Y_2$ regardless of n, P, or γ which is, of course, disturbing. Actually we know only that $5950 \leq Y_1 \leq Y_2 \leq 6050$. A conservative approach would be to consider the worst case situation, i.e. $Y_1 = 5950$ and $Y_2 = 6050$. In this case the limit is given as

$$\begin{aligned} L''_H &= 6050 - b_H(6050-5950) \\ &= 6050 - 28.38\ (100) \\ &= 3212. \end{aligned}$$

Upon inspection of the data we see that the 72 test values ranged only from 6000 to 6900 which makes the lower tolerance limit of 3212 seem excessively low. Another approach would be to consider the two pipe pressures which were rounded to 6000 to be uniformly spaced on the interval (5950,6050), i.e. $Y_1 = 5983.33$ and $Y_2 = 6016.67$. Using this approach

$$\begin{aligned} L'_H &= 6016.67 - 28.38(6016.67 - 5983.33) \\ &= 5071 \end{aligned}$$

a more intuitively appealing result. However, one might be concerned that the limits obtained using this method would not actually be true 1-P content tolerance limits with at least confidence γ. We will address this problem in the next section.

When using the tabled results of Hanson and Koopmans, the dispersion and location of the distribution is assessed by means of two adjacent order statistics. Intuitively, when the measurements are crude, the information given by successive order statistics concerning the dispersion, is greatly diminished and can be misleading. More appealing limits would deal with non-adjacent order statistics in order to provide a more accurate measure of variability.

In light of these considerations, an intuitively appealing limit would be that for which $k = 0$ and $j = n - 1$, i.e. when the tolerance limit depends on the range. The lower and upper tolerance limits are then respectively

$$L_R = Y_n - b_R(Y_n - Y_1) \tag{5}$$

and

$$U_R = Y_1 + b_R\,(Y_n - Y_1). \tag{6}$$

(Table 2 here)

Table 2 presents values of b_R for various values of n, P, and γ. For a discussion of the computations involved in evaluating b_R see Appendix A.

It should be noted that for a given P and γ, the tolerance limits given in (5) and (6) as well as (3) and (4) collapse to the corresponding traditional nonparametric tolerance limits whenever n is greater than or equal to the minimum sample size required for the nonparametric limits to exist. This minimum sample size for each set of parameters in Table 2 is given in parentheses following the last tolerance factor.

Applying (5) to the data of Table 1 and assuming the worst case situation, i.e. $Y_1 = 5950$ and $Y_n = 6950$, we obtain

$$\begin{aligned} L''_R &= 6950 - b_R(6950 - 5950) \\ &= 6950 - 1.658(1000) \\ &= 5292 \end{aligned}$$

whereas assuming a uniform spacing yields

$$\begin{aligned} L'_R &= 6900 - 1.658(6900-5983.33) \\ &= 5380. \end{aligned}$$

In this example the limits based on the range gave lower tolerance limits which were greater than those based on the adjacent order statistics for both methods of dealing with rounded data. In Table 3 tolerance limits

calculated by the methods of this section are presented for eleven grades of pipe which were tested. Of the eleven grades of pipe, the L'_R limits

(Table 3 here)

were greater than the L'_H limits eight times, and the L''_R were greater than the L''_H ten times. Indeed some of the L_H limits are very poor, e.g. grades 3 and 11. These limits are poor whether the worst case method or the uniform spacing method for handling ties is used. Also of interest is the fact that for the limits based on the range, the choice of method made less difference than it did for limits based on adjacent order statistics. It should be noted that for $1 - P = .995$ and $\gamma = .95$, the standard nonparametric limits do not exist for $n < 598$ and thus are not applicable here.

Monte Carlo Comparison of Tolerance Limits

In this section Monte Carlo comparisons of the tolerance limits based on the range and on adjacent order statistics will be discussed. As a first comparison these tolerance limits are compared using "exact" data. In Table 4 the results of these comparisons for the normal, exponential, and chi-square distributions are given. These comparisons were made for various values of P and n. All runs were at the nominal $\gamma = .95$ level and the $\hat{\gamma}$ given in the table is the estimate of γ based on 1000 repetitions. The quantities $\hat{\mu}$ and $\hat{\sigma}$ are estimates of the mean and variance of the tolerance limits. The order statistics were generated using the method of Schucany [4].

(Table 4 here)

In order to compare the tolerance limits discussed in this section, a method of comparison needs to be specified. Goodman and Madansky [1] have suggested that one-sided lower (upper) limit A_1 is better than A_2 if $E[A_1 - A_2] > 0$ (<0). We will employ this criterion to our situation also.

With this in mind the following observations concerning Table 4 are made:

(a) L_R and U_R are superior for the normal distribution while U_R is superior for the right tails of the distributions skewed to the right as would be expected. For distributions strongly skewed to the right such as the exponential or chi-square with small degrees of freedom, the L_H limits tended to be superior.

(b) The superiority of L_R and U_R for the normal distribution and right tails of the skewed distributions is greater for the smaller sample sizes, i.e. when b_H is quite large.

(c) L_R and U_R are in general less variable than L_H and U_H.

(d) Although L_R and U_R in general show to be more conservative in the sense that their $\hat{Y}$'s are larger, this conservatism is often accompanied by superior limits using the Goodman and Madansky criterion. Of course this apparent contradiction occurs because of the lower variability of the L_R and U_R limits.

It should be noted that neither type of tolerance limit performed well for distributions such as the beta and uniform with known and finite support. In fact for these distributions, sample sizes, and parameters employed in Table 4, $\hat{\mu}$ fell outside the support in most cases.

The results of Table 4 indicate that the limits based on the range which were developed to deal with data of limited accuracy are superior in some cases to the limits based on successive order statistics even when data is "exact."

A second Monte Carlo comparison was performed to compare the tolerance limits based on the range and those based on adjacent order statistics when data is of limited accuracy. In Appendix B a formulation of the uniform spacing method of dealing with rounded data is given.

In Table 5 results of the Monte Carlo examination of this method of dealing with rounded data are given. From the table we see that there is close agreement between limits of Table 5 and corresponding limits of Table 4. In addition there were no cases of confidence γ being small enough for us to reject the null hypothesis that $\gamma \geq .95$ at the .05 level. For these reasons we feel that the uniform spacing method for handling rounded data is a good one and thus that the worst case method is unnecessarily conservative.

(Table 5 here)

Summary

In obtaining tolerance limits, the engineer might use, for example, the procedures outlined in MIL-HDBK-5C (see [3]); and there he is presented with the alternative of using the standard nonparametric procedure if "near normality" cannot be demonstrated. As discussed previously, sample size can be a problem when using the standard nonparametric techniques. For example, an A-basis ($P = .01$, $\gamma = .95$) distribution free tolerance limit requires a sample of size 296. When such a sample size cannot be obtained due to practical considerations, the normality assumption may be invoked out of necessity. In this paper we have applied theory due to Hanson and and Koopmans [2], which is not well known, to present an alterative course of action.

Whereas Hanson and Koopmans applied their theory using adjacent order statistics, we have considered another utilization of that theory which involves the range. Results presented in this paper show that these limits involving the range do have merit. We have compared the two techniques in this paper and have outlined recommendations for their use. The decision between the two utilizations of the Hanson and Koopmans theory may also involve nonstatistical considerations. For example if a lower tolerance limit is desired when items are placed on simultaneous test, then a savings in time will be obtained by forming the limit using the first two order statistics.

When computing the limits, one may also be faced with the problem of tied observations such as those given in Table 1 concerning collapse pressure. A procedure (the uniform spacing method) for handling this situation is presented.

As equations (1) and (2) indicate, the tolerance limits may be based on any combination of order statistics. The main problem is in solving equation (A2). For example, lower tolerance limits based on the first and $[\frac{n+1}{2}]$ order statistics might prove effective if the parent distribution is skewed to the right.

Appendix A

Summary of Hanson and Koopmans Results

In [2], Hanson and Koopmans developed the theory which provides the basis for calculation of tolerance limits, for any sample size, in the following two situations:

(a) If F is the distribution function, then when $\log F$ is concave it is possible to obtain lower tolerance limits. In this case the lower tolerance limit is given by

$$L = Y_{k+j+1} - b(Y_{k+j+1} - Y_{k+1}) \tag{A1}$$

where Y_m denotes the m^{th} order statistic and the value of b is such that

$$\pi(b) = \frac{n!}{(n-k-j-1)!\,(j-1)!\,k!}\left\{\int_0^p\int_0^v + \int_p^1\int_0^{p^{1/b}v^{(b-1)/b}}\right\} w^k (v-w)^{j-1}(1-v)^{n-k-j-1}\,dw\,dv \tag{A2}$$

$= \gamma$, whenever $\pi(1) < \gamma$.

When $\pi(1) \geq \gamma$, the value of b is taken to be unity, i.e. $L = Y_{k+1}$.

(b) When $\log(1-F)$ is concave, then upper tolerance limits can be calculated. An equivalent condition is that the density function have an increasing hazard rate. In this case the upper tolerance limit is given by

$$U = Y_{n-k-j} + b(Y_{n-k} - Y_{n-k-j}) \tag{A3}$$

where b is as in (A2). As before when $\pi(1) \geq \gamma$, the value of b is taken to be unity, i.e. $U = Y_{n-k}$.

Of course if the underlying lifetime distribution is such that both $\log F$ and $\log(1-F)$ are concave then either lower or upper tolerance limits may be obtained using the method of Hanson and Koopmans. Indeed those authors pointed out that there is an important class of distributions for which both $\log F$ and $\log(1-F)$ are concave. Examples of members of this

class are the normal, gamma, beta and Weibull distributions --- either in truncated or original form. Thus the class includes distributions possessing density functions which are quite often employed to describe lifetime situations.

Hanson and Koopmans [2] have tabled the constant b for various values of γ, P, and sample size n for the case in which $j = 1$ in (A1) and (A3). In this case they were able to reduce the double integral in (A2) to a sum of two one-dimensional integrals. In fact they were able to show that in this case, $\pi(b)$ could be expressed in terms of the gamma function and the incomplete beta distribution function which enabled existing computer routines to be used in the evaluation of b.

In the present paper we investigate the evaluation of $\pi(b)$ when $j = n - 1$. In this case (A2) reduces to the one dimensional integral

$$\pi(b) = 1 - n\int_{P}^{1} v^{n-1}\left[1 - \left(\frac{P}{v}\right)^{1/b}\right]^{n-1} dv. \tag{A4}$$

The integration involved in evaluating

$$g(b) = \pi(b) - \gamma \tag{A5}$$

in this case was performed with 20 point Gauss-Legendre quadrature and roots of (A5) were approximated by the method of false position. Checks were made for various values of the parameters using a series solution to the integral in (A4) in lieu of employing Gaussian quadrature. At least five decimal place agreement was observed in all cases checked. All calculations were performed on the CDC Cyber 72 computer. Values of b in this case with $j = n - 1$ are presented in Table 2 for various values of γ, P, and n.

Appendix B

Uniform Spacing Method of Calculating Tolerance Limits when Data is of Limited Accuracy

Suppose that lower tolerance limits are desired for a continuous theoretical lifetime distribution for which the limited accuracy of the measuring device has resulted in the observed order statistics $z_1, \ldots, z_n$, whereas had the measurements been "perfect" the order statistics would have been $y_1, \ldots, y_n$, respectively. Assuming that the only measurement errors made are those due to round-off, then if the measuring device is accurate to the nearest r, it is known only that for any m, $z_m - \frac{r}{2} \le y_m \le z_m + \frac{r}{2}$. Of course r is always present to some degree when measurements are being made on continuous variables. Application of the Hanson and Koopmans method would yield as the lower tolerance limit

$$L_z = z_{k+j+1} - b(z_{k+j+1} - z_{k+1}). \tag{B1}$$

The theoretical limit is given by

$$L = y_{k+j+1} - b(y_{k+j+1} - y_{k+1}) \tag{B2}$$

which may vary from L_z in (B1) by as much at $\pm$ (b-.5)r.

Assume that the order statistics $z_1, \ldots, z_n$ have been observed, and let $w_1 = z_1$. Suppose further that because of the imprecision of the measuring device, the only possible observable values for $z_2, z_3, \ldots, z_n$ are w_1, $w_1 + r = w_2$, $w_2 + r = w_3$, etc. Now suppose that n_1 of the z_i's are equal to w_1, n_2 equal to w_2, ..., and n_k equal to $w_k = z_n$. We have two cases for approximating y_1 and y_2:

(1) Suppose $z_2 = w_1$, i.e. $n_1 \ge 2$. Then our approximations of the unknown y_1 and y_2 are given by $\hat{y}_1$ and $\hat{y}_2$, the expected values of the first two ordered uniform variables over $(z_1 - \frac{r}{2}, z_1 + \frac{r}{2})$ in a sample size of n_1 i.e. $\hat{y}_1 = z_1 - \frac{r}{2} + \frac{r}{n_1+1}$ and $\hat{y}_2 = \hat{y}_1 + \frac{r}{n_1+1}$.

(2) Suppose $n_1 = 1$ (assume $n_2 > 0$). Then following the same reasoning, $\hat{y}_1$ is the expected value of the first ordered uniform over $(z_1 - \frac{r}{2}, z_1 + \frac{r}{2})$ in a sample of size 1, i.e. $\hat{y}_1 = z_1$. Likewise, $\hat{y}_2$ is the first ordered uniform over $(z_2 - \frac{r}{2}, z_2 + \frac{r}{2})$ in a sample of size n_2, i.e.

$$\hat{y}_2 = z_2 - \frac{r}{2} + \frac{r}{n_2+1} .$$

Approximations of y_n and y_{n-1} are obtained in a similar manner. Using these approximations, observed values of the tolerance limits L'_H, U'_H, L'_R, and U'_R corresponding to (3),(4),(5), and (6) respectively are obtained by substituting $\hat{y}_1$, $\hat{y}_2$, $\hat{y}_{n-1}$, and $\hat{y}_n$ for Y_1, Y_2, Y_{n-1}, and Y_n respectively.

Of course distributions other than the uniform could be considered over the intervals $(z_i - \frac{r}{2}, z_i + \frac{r}{2})$ but the authors feel that this additional sophistication in the technique would add no meaningful improvement.

ACKNOWLEDGMENT

The authors would like to express their appreciation to a referee for his many helpful suggestions.

REFERENCES

1. Goodman, Leo A., and Madansky, Albert, "Parameter-Free and Non-parametric Tolerance Limits: The Exponential Case," Technometrics, Vol. 4, No. 1, February, 1962, pp. 75-95.

2. Hanson, D.L., and Koopmans, L.H., "Tolerance Limits for the Class of Distributions with Increasing Hazard Rates," Annals of Mathematical Statistics, Vol. 35, No. 4, December, 1964, pp. 1561-1570.

3. Military Standards Handbook, "Metallic Materials and Elements of Aerospace Vehicle Structures," MIL-HDBK-5C, Section 9, United States Government Printing Office, December, 1978.

4. Schucany, W.R., "Order Statistics and Simulation," Journal of Statistical Computation and Simulation, Vol. 1, 1972, pp. 281-286.

5. Walpole, Ronald E. and Myers, Raymond H., Probability and Statistics for Engineers and Scientists, 2nd ed., Macmillan, New York, 1978.

Table 1 Collapse Pressures for a Sample of Pipes--Grade 1

Pressure	Frequency
6000	2
6100	6
6200	18
6300	7
6400	12
6500	6
6600	9
6700	5
6800	6
6900	1
	72

TABLE 2

Tolerance Factors b_R for Tolerance Limits Depending on the Range

n	γ = .95	γ = .99
	P = .10	
2	35.17691	179.79060
3	7.65870	18.92398
4	4.50521	8.55170
5	3.30616	5.58036
6	2.69502	4.23786
7	2.32317	3.48252
8	2.07170	2.99948
9	1.88932	2.66374
10	1.75034	2.41636
11	1.64046	2.22610
12	1.55110	2.07488
13	1.47677	1.95154
14	1.41381	1.84883
15	1.35966	1.76180
16	1.31251	1.68700
17	1.27100	1.62193
18	1.23412	1.56472
19	1.20108	1.51397
20	1.17128	1.46859
21	1.14423	1.42773
22	1.11954	1.39071
23	1.09688	1.35698
24	1.07601	1.32610
25	1.05670	1.29770
26	1.03877	1.27148
27	1.02207	1.24717
28	1.00645	1.22456
29	(29)	1.20347
30		1.18374
31		1.16523
32		1.14783
33		1.13142
34		1.11593
35		1.10126
36		1.08736
37		1.07415
38		1.06159
39		1.04962
40		1.03820
41		1.02728
42		1.01684
43		1.00684
		(44)

n	γ = .95	γ = .99
	P = .05	
2	48.63158	248.39916
3	10.57547	25.44104
4	6.00391	11.38610
5	4.38315	7.39232
6	3.56125	5.59617
7	3.06293	4.58871
8	2.72681	3.94504
9	2.44356	3.49906
10	2.29852	3.17184
11	2.15243	2.91978
12	2.03377	2.71965
13	1.93518	2.55656
14	1.85174	2.42085
15	1.78005	2.30595
16	1.71767	2.20725
17	1.66278	2.12143
18	1.61405	2.04602
19	1.57042	1.97916
20	1.53108	1.91940
21	1.49539	1.86560
22	1.46283	1.81688
23	1.43298	1.77251
24	1.40548	1.73190
25	1.38004	1.69456
26	1.35643	1.66009
27	1.33444	1.62815
28	1.31389	1.59845
29	1.29464	1.57075
30	1.27655	1.54484
31	1.25952	1.52054
32	1.24345	1.49770
33	1.22824	1.47617
34	1.21383	1.45583
35	1.20015	1.43659
36	1.18714	1.41836
37	1.17474	1.40104
38	1.16292	1.38456
39	1.15162	1.36887
40	1.14081	1.35389
41	1.13046	1.33959

TABLE 2--Continued

n	γ .95	γ .99
	P = .05	
42	1.12053	1.32590
43	1.11100	1.31279
44	1.10184	1.30022
45	1.09303	1.28816
46	1.08454	1.27656
47	1.07636	1.26541
48	1.06846	1.25467
49	1.06084	1.24432
50	1.05348	1.23434
52	1.03946	1.21540
54	1.02631	1.19768
56	1.01394	1.18108
58	1.00228	1.16548
60		1.15077
62	(59)	1.13689
64		1.12375
66		1.11129
68		1.09946
70		1.08820
72		1.07747
74		1.06723
76		1.05744
78		1.04807
80		1.03909
82		1.03047
84		1.02220
86		1.01424
88		1.00658
		(90)

n	γ .95	γ .99
	P = .01	
2	80.00380	406.43565
3	16.91220	40.65001
4	9.49579	17.09305
5	6.89049	11.61292
6	5.57681	8.75814
7	4.78352	7.16267
8	4.25011	6.14747
9	3.86502	5.44459
10	3.57267	4.92834
11	3.34227	4.53234
12	3.15540	4.21831
13	3.00033	3.96268
14	2.86924	3.75016
15	2.75672	3.57038
16	2.65889	3.41607
17	2.57290	3.28198
18	2.49660	3.16423
19	2.42833	3.05988
20	2.36683	2.96666
21	2.31106	2.88279
22	2.26020	2.80687
23	2.21359	2.73775
24	2.17067	2.67451
25	2.13100	2.61638
26	2.09419	2.56274
27	2.05991	2.51305
28	2.02790	2.46687
29	1.99791	2.42380
30	1.96975	2.38353
31	1.94324	2.34576
32	1.91822	2.31027
33	1.89457	2.27683
34	1.87215	2.24525
35	1.85088	2.21538
36	1.83065	2.18707
37	1.81134	2.16019
38	1.79301	2.13462
39	1.77546	2.11027
40	1.75868	2.08704
41	1.74260	2.06485

TABLE 2--Continued

n	γ = .95	γ = .99
	P = .01	
42	1.72718	2.04363
43	1.71239	2.02331
44	1.69817	2.00382
45	1.68449	1.98512
46	1.67132	1.96715
47	1.65862	1.94986
48	1.64638	1.93322
49	1.63456	1.91719
50	1.62313	1.90172
52	1.60139	1.87238
54	1.58101	1.84495
56	1.56184	1.81924
58	1.54377	1.79509
60	1.52670	1.77233
62	1.51053	1.75084
64	1.49520	1.73051
66	1.48063	1.71124
68	1.46675	1.69293
70	1.45352	1.67552
72	1.44089	1.65892
74	1.42881	1.64309
76	1.41724	1.62795
78	1.40614	1.61347
80	1.39549	1.59959
82	1.38525	1.58627
84	1.37541	1.57348
86	1.36592	1.56119
88	1.35678	1.54936
90	1.34796	1.53796
92	1.33944	1.52696
94	1.33120	1.51635
96	1.32324	1.50611
98	1.31553	1.49620
100	1.30806	1.48662
105	1.29036	1.46396
110	1.27392	1.44297
115	1.25859	1.42346
120	1.24425	1.40526
125	1.23080	1.38822
130	1.21814	1.37223
135	1.20620	1.35718
140	1.19491	1.34298
145	1.18421	1.32955
150	1.17406	1.31682
155	1.16440	1.30473
160	1.15519	1.29324
165	1.14640	1.28228
170	1.13801	1.27183
175	1.12997	1.26184
180	1.12226	1.25228
185	1.11486	1.24311
190	1.10776	1.23431
195	1.10092	1.22586
200	1.09434	1.21774
205	1.08799	1.20991
210	1.08187	1.20237
215	1.07595	1.19509
220	1.07024	1.18807
225	1.06471	1.18128
230	1.05935	1.17471
235	1.05417	1.16836
240	1.04914	1.16220
245	1.04426	1.15623
250	1.03952	1.15044
275	1.01773	1.12388
300	(299)	1.10067
325		1.08014
350		1.06179
375		1.04526
400		1.03025
425		1.01653
450		1.00392
		(459)

TABLE 2--Continued

	γ			γ	
n	.95	.99	n	.95	.99
	P = .005			P = .005	
2	93.52083	477.40521	42	1.98847	2.35276
3	19.64593	47.21468	43	1.97140	2.32933
4	11.00178	20.84466	44	1.95500	2.30686
5	7.97155	13.43315	45	1.93923	2.28530
6	6.44564	10.12147	46	1.92404	2.26458
7	5.52508	8.27825	47	1.90940	2.24466
8	4.90656	7.09637	48	1.89528	2.22548
9	4.46029	6.28266	49	1.88165	2.20699
10	4.12166	5.68528	50	1.86848	2.18917
11	3.85491	5.22720	52	1.84342	2.15534
12	3.63882	4.86405	54	1.81991	2.12373
13	3.45920	4.56852	56	1.79781	2.09410
14	3.30757	4.32288	58	1.77699	2.06626
15	3.17745	4.11513	60	1.75731	2.04003
16	3.06435	3.93684	62	1.73868	2.01527
17	2.96495	3.78194	64	1.72100	1.99184
18	2.87676	3.64594	66	1.70421	1.96963
19	2.79788	3.52543	68	1.68822	1.94854
20	2.72682	3.41779	70	1.67297	1.92847
21	2.66239	3.32096	72	1.65841	1.90935
22	2.60365	3.23331	74	1.64449	1.89110
23	2.54982	3.15353	76	1.63115	1.87366
24	2.50026	3.08054	78	1.61837	1.85696
25	2.45446	3.01346	80	1.60610	1.84099
26	2.41196	2.95155	82	1.59430	1.82565
27	2.37239	2.89422	84	1.58295	1.81092
28	2.33543	2.84092	86	1.57203	1.79675
29	2.30082	2.79123	88	1.56149	1.78312
30	2.26832	2.74477	90	1.55133	1.76999
31	2.23772	2.70121	92	1.54151	1.75732
32	2.20886	2.66026	94	1.53203	1.74510
33	2.18156	2.62169	96	1.52285	1.73330
34	2.15570	2.58527	98	1.51397	1.72189
35	2.13115	2.55082	100	1.50537	1.71085
36	2.10782	2.51817	105	1.48498	1.68475
37	2.08559	2.48716	110	1.46604	1.66058
38	2.06440	2.45768	115	1.44838	1.63811
39	2.04415	2.42960	120	1.43187	1.61715
40	2.02479	2.40282	125	1.41637	1.59753
41	2.00625	2.37723	130	1.40179	1.57911

TABLE 2--Continued

n	γ .95	γ .99
	P = .005	
135	1.38804	1.56178
140	1.37504	1.54542
145	1.36272	1.52996
150	1.35102	1.51530
155	1.33990	1.50139
160	1.32930	1.48815
165	1.31918	1.47554
170	1.30951	1.46350
175	1.30025	1.45200
180	1.29138	1.44099
185	1.28286	1.43043
190	1.27468	1.42031
195	1.26681	1.41058
200	1.25923	1.40122
205	1.25192	1.39221
210	1.24487	1.38353
215	1.23806	1.37515
220	1.23148	1.36706
225	1.22511	1.35925
230	1.21895	1.35169
235	1.21298	1.34437
240	1.20719	1.33728
245	1.20157	1.33041
250	1.19611	1.32375
275	1.17103	1.29318
300	1.14902	1.26646
325	1.12940	1.24282
350	1.11199	1.22171
375	1.09617	1.20268
400	1.08177	1.18540
425	1.06858	1.16961
450	1.05644	1.15510
475	1.04520	1.14170
500	1.03476	1.12927
525	1.02501	1.11769
550	1.01589	1.10687
575	1.00733	1.09672
600	(598)	1.08718
625		1.07818
650		1.06967
675		1.06161
700		1.05395
725		1.04666
750		1.03972
775		1.03309
800		1.02675
825		1.02068
850		1.01486
875		1.00927
900		1.00389
		(919)

Table 3 Comparison of Lower Tolerance Limits with $1-P = .995$ and $\gamma = .95$ for Various Grades of Pipe

GRADE	n	L'_H	L'_R	L''_H	L''_R	Z_1	Z_2	Z_n
1	72	5071	5380	3212	5292	6000	6000	6900
2	83	6035	7324	3094	7225	8100	8200	9400
3	54	-6770	7404	-9994	7272	9700	10200	12500
4	79	4932	6398	2314	6287	7500	7600	9300
5	58	8903	7795	6810	7697	10000	10000	12800
6	51	11256	8346	8969	8241	12400	12400	17100
7	54	9252	12028	5928	11896	15600	16000	20400
8	57	3782	3768	1378	3684	4600	4600	5600
9	99	4179	5239	1428	5138	5800	5900	6900
10	84	5353	6526	2438	6417	7400	7500	8900
11	51	-3743	4859	-7174	4724	6400	6700	8200

Table 4 - Monte Carlo Results

Normal (0,1), γ = .95 , 1000 repetitions

		L_H			L_R			U_H'			U_R		
1-P	n	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$
.99	50	-10.85	.264	.962	-5.06	.026	1.000	10.40	.254	.946	5.04	.026	1.000
.99	100	- 4.15	.059	.908	-4.02	.017	1.000	4.28	.064	.918	4.05	.018	1.000
.95	10	-12.37	.323	.951	-5.55	.047	1.000	12.61	.313	.957	5.56	.047	1.000
.95	25	- 4.85	.094	.950	-3.45	.023	.999	4.91	.092	.960	3.46	.022	1.000
.95	50	- 2.54	.021	.947	-2.50	.016	.976	2.54	.022	.951	2.50	.016	.973
.90	10	- 6.59	.151	.961	-3.89	.034	.998	6.35	.138	.950	3.85	.034	.997
.90	25	- 2.22	.021	.957	-2.20	.016	.981	2.24	.022	.955	2.21	.017	.974

Exponential (1) γ = .95, 1000 repetitions

		L_H			L_R			U_H			U_R		
1-P	n	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$
.99	50	- .42	.014	.968	-2.78	.025	1.000	25.75	.694	.955	7.32	.066	.948
.99	100	- .04	.002	.942	-1.58	.012	1.000	10.15	.184	.926	6.75	.051	.957
.95	10	- 2.15	.069	.965	-3.60	.053	1.000	23.07	.700	.931	6.64	.093	.950
.95	25	- 0.23	.009	.967	-1.41	.015	1.000	10.86	.249	.952	5.31	.055	.961
.95	50	0.06	.001	.959	-0.22	.002	1.000	5.14	.057	.955	4.69	.040	.953
.90	10	- 0.94	.035	.976	-2.03	.029	1.000	11.92	.313	.945	5.06	.067	.955
.90	25	0.16	.001	.955	-0.18	.003	.999	4.44	.058	.955	4.09	.045	.956

Table 4 - Continued

Chi-Square (5), γ = .95, 1000 repetitions

		L_H			L_R			U_H			U_R		
1-P	n	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\gamma}$
.99	50	- 5.89	.175	.964	- 8.21	.064	1.000	70.89	1.767	.955	23.81	.161	.992
.99	100	- 0.62	.033	.931	- 4.43	.031	1.000	28.78	.471	.924	21.40	.128	.990
.95	10	-14.10	.420	.978	-10.76	.135	1.000	63.22	1.610	.946	22.93	.226	.987
.95	25	- 1.96	.084	.961	- 3.68	.042	1.000	30.29	.625	.954	17.65	.138	.973
.95	50	0.48	.013	.941	- 0.64	.012	.998	16.57	.155	.942	15.55	.109	.953
.90	10	- 5.54	.190	.964	- 5.55	.080	1.000	33.80	.768	.950	17.62	.173	.978
.90	25	0.67	.017	.958	0.21	.016	.993	14.70	.143	.943	13.94	.110	.954

Table 5 - Tolerance Limits with Rounded Data $\left(r = \frac{\text{STD DEV}}{2}\right)$

Normal (0,1) , γ = .95, 1000 repetitions

		L'_H		L'_R		U'_H		U'_R	
1-P	n	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$
.99	50	-10.8	1.00	-5.1	1.00	10.9	1.00	5.2	1.00
.99	100	- 4.5	.98	-4.2	1.00	4.3	.97	4.1	1.00
.95	10	-13.1	1.00	-5.7	1.00	12.7	1.00	5.7	1.00
.95	25	- 5.1	1.00	-3.5	1.00	5.0	.99	3.5	1.00
.95	50	- 2.6	.97	-2.5	.98	2.6	.97	2.5	.98
.90	10	- 6.8	.99	-3.9	1.00	6.4	1.00	3.9	1.00
.90	25	- 2.2	.99	-2.2	.99	2.3	.99	2.3	.99

Exponential (1) , γ = .95, 1000 repetitions

		L'_H		L'_R		U'_H		U'_R	
1-P	n	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$
.99	50	- 0.5	1.00	-2.8	1.00	26.2	1.00	7.3	.95
.99	100	- 0.0	1.00	-1.6	1.00	10.4	.95	6.9	.95
.95	10	- 2.3	1.00	-3.6	1.00	23.3	.99	6.6	.94
.95	25	- 0.3	1.00	-1.4	1.00	10.5	.98	5.2	.97
.95	50	0.0	1.00	-0.2	1.00	5.5	.94	4.9	.95
.90	10	- 1.0	1.00	-2.0	1.00	12.3	.99	5.2	.94
.90	25	0.0	1.00	-0.2	1.00	4.4	.97	4.0	.97

Table 5 - Continued

Chi-Square (5) , γ = .95, 1000 repetitions

		L'_H		L'_R		U'_H		U'_R	
1-P	n	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$	$\hat{\mu}$	$\hat{\gamma}$
.99	50	- 6.0	1.00	- 8.6	1.00	67.0	1.00	23.9	.99
.99	100	- 0.9	1.00	- 4.8	1.00	29.0	.94	21.8	.99
.95	10	-15.8	1.00	-11.1	1.00	66.7	1.00	23.0	.99
.95	25	- 2.6	1.00	- 4.0	1.00	29.7	.99	17.8	.99
.95	50	0.2	1.00	- 0.4	1.00	16.6	.99	15.7	.97
.90	10	- 6.4	1.00	- 6.0	1.00	35.1	.99	18.2	.99
.90	25	0.4	.97	0.0	.99	14.4	.98	13.7	.98

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 135

2. GOVT ACCESSION NO.:

3. RECIPIENT'S CATALOG NUMBER:

4. TITLE (and Subtitle): ONE-SIDED TOLERANCE LIMITS FOR A BROAD CLASS OF LIFETIME DISTRIBUTIONS WITH APPLICATIONS TO DATA OF LIMITED ACCURACY

5. TYPE OF REPORT & PERIOD COVERED: Technical Report

6. PERFORMING ORG. REPORT NUMBER: 135

7. AUTHOR(s): Wayne A. Woodward & William H. Frawley

8. CONTRACT OR GRANT NUMBER(s): N00014-75-C-0439

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Southern Methodist University, Dallas, TX 75275 & E-Systems, Incorporated, Greenville, TX 75401

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR 042 280

11. CONTROLLING OFFICE NAME AND ADDRESS: OFFICE OF NAVAL RESEARCH ARLINGTON, VA. 22217

12. REPORT DATE: July 25, 1979

13. NUMBER OF PAGES: 27

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):

15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited. Reproduction in whole or in part is permiteed for any purpose of the United States Government.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Tolerance limits, distribution free, increasing hazard rate

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Addressed is the problem of determining a one-sided tolerance limit for a population possessing a distribution belonging to a broad class of lifetime distributions. A new implementation of existing general theory is given and contrasted with an earlier utilization of that theory. General guidelines are given for deciding which implementation to use. A method for adjusting for the accuracy of the measuring device is discussed and illustrated with an actual example.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)